Pertanika J. Sci. & Technol. 22 (1): 15 - 24 (2014)

PERTANIKA JOURNALS

SCIENCE & TECHNOLOGY

Journal homepage: http://www.pertanika.upm.edu.my/

Short Communication

FRFT Based Timing Estimation Method for an OFDM System

Saxena, R.[1] and Joshi, H. D.[2]*

[1]*Director, Jaypee University, Anoopshahr, District Bulandshahr– 203390 (U.P.), India*
[2]*Department of ECE, Thapar University, Patiala – 147001 (Punjab), India*

ABSTRACT

The Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation (MCM) technique which is adopted by many wireless communication standards for transmitting data at very high rates over time dispersive radio channels. In an OFDM system, the timing estimation is extremely important for maintaining orthogonality among the subcarriers. In this paper, a method of timing estimation is proposed for an OFDM system. The proposed method used chirp signal as a training sequence and employed the fractional Fourier transform (FRFT) as a tool to localize the training sequence (chirp) at the receiver. The comparative study showed the superiority of the proposed estimator in terms of mean and MSE of timing offset. The MSE of timing offset with proposed method was found to be 76% (5 dB SNR) and 63% (8 dB SNR) lower than Awoseyila *et al.'s* method in *HIPERLAN/2 indoor channel-A* and in *Wi-Max system* (strong fading channel), respectively. However, the improvement in MSE is obtained in the proposed method at the cost of increased computational complexity, in terms of $\frac{N}{4} - 1 + N\log_2 N$ more complex multiplication than the Awoseyila *et al.'s* method.

Keywords: Timing Estimation, OFDM, FRFT, Chirp

INTRODUCTION

The Orthogonal Frequency Division Multiplexing (OFDM) has drawn major attention over the last decade for its usefulness in broad band wireless communication. Due to advantageous features such as high spectral efficiency, robustness to fading channel and easy equalization, the OFDM has been adopted as a major data transmission technique by many wireless communication standards such as IEEE 802.11a, IEEE 802.16a and terrestrial digital video broadcasting (DVB-T) systems (Keller & Hanzo, 2000).

Article history:
Received: 1 November 2011
Accepted: 20 February 2012

E-mail addresses:
srajiv2008@gmail.com (Saxena, R.),
hemdutt@gmail.com (Joshi, H. D.)
*Corresponding Author

ISSN: 0128-7680

However, one of the major disadvantages of OFDM system is its sensitivity to synchronization errors (time and frequency) at the receiver. Inaccurate estimation of timing and carrier frequency offset destroys the orthogonality among sub-carrier, which generates inter-carrier interference (ICI) and inter-block interference (IBI) (Morelli *et al*., 2007). Several methods which are based on the transmission of known preamble have been proposed in the past for correct timing and frequency estimation, either jointly or individually (Schmidl & Cox, 1997; Minn *et al*., 2000; Park *et al*., 2003; Shi & Serpedin, 2004; Ren *et al*., 2005; Awoseyila *et al*., 2008). The basic concept of these methods is to transmit a preamble which consists of some repetitive blocks and then, applying a sliding window correlator at the receiver to detect the maximum of timing metric. The timing metric with a sharp peak is desired for better timing estimate.

In the method by Schmidl and Cox (S&C) (1997), the timing metric exhibits a large plateau which causes a large variance in timing estimation. In order to reduce the variance, more algorithms are given by (Minn *et al*., 2000; Park *et al*., 2003; Shi & Serpedin, 2004; Ren *et al*., 2005; Awoseyila *et al*., 2008; Boumard & Mammela, 2009) with sharper timing metric and less variance. In this paper, a new timing estimation method is proposed which is based on localization of chirp signal with the help of fractional Fourier transform (FRFT).

In the proposed estimator, the emphasis is on the use of fractional Fourier transform based correlation and chirp signal. The application of chirp signal for the synchronization in OFDM system along with its advantages is very well documented by Boumard and Mammela (2009). The chirp signals have characteristics such as finite duration, finite bandwidth and better auto-correlation property. The FRFT is a generalization of the conventional Fourier transform in time-frequency plane and has found applications in non-stationary signals (chirp signal) analysis, especially in filtering, time delay estimation and radar signal processing (Almeida, 1994; Sun *et al*., 2002; Sharma & Joshi, 2007; Tao *et al*., 2009; Singh & Saxena, 2011). It is due to the fact that the FRFT nicely localizes the chirp signal in time-frequency plane (Tao *et al*., 2009). This characteristic of FRFT on chirp signal has been exploited in the proposed estimator.

Various properties of the FRFT have already been derived and established, as given by Almeida (1994). Recently, the new weighted convolution and correlation theorems in the FRFT domain have been given by Singh *et al*. (2011). The correlation theorem given by Singh and Saxena (2011) is used in the proposed timing estimation method.

The performance of proposed timing estimator is compared with the algorithms given by Schmidl and Cox (1997), Minn *et al.* (2000), Park *et al.* (2003), Shi and Serpedin (2004) and Awoseyila *et al.* (2008). It was found that the mean and mean-square error (MSE) of timing offset with proposed algorithm was better than other algorithms as mentioned above, both in ***HIPERLAN/2 indoor channel-A*** (Channel Models, 1998) *and* in strong fading channel (Wi-Max with 256 sub-carriers).

SYSTEM MODEL

An OFDM system with N sub-carrier has been considered for the present analysis. The received signal $y(n)$ from a multipath fading channel of memory length L may be represented as:

$$y(n) = \sum_{m=0}^{L-1} h(m)\, s(n-m) \ , \quad n = 0,1,2......N-1 \tag{1}$$

where, $h(m)$ is the channel impulse response of mth path and $s(n)$ is the transmitted time domain OFDM signal expressed by:

$$s(n) = \sum_{k=0}^{N-1} d_k\, e^{j2\pi k\, n/N},\ n = 0,1,2......N-1 \tag{2}$$

where, d_k is the complex data symbol modulated on the kth sub-carrier. At the receiver, timing offset is considered as a delay in the received signal and frequency offset is taken as phase distortion of received data in time domain. After considering the effects of timing offset, frequency offset, and AWGN noise in the received signal, the y(n) given by (1) can be re-written as:

$$r(n) = y(n-\tilde{d})\, e^{j2\pi\varepsilon n/N} + w(n) \tag{3}$$

where, $\tilde{d}$ is the integer-valued unknown arrival time of a symbol, 'ε' is the frequency offset normalized by the sub-carrier spacing ($1/NT_s$), $w(n)$ is a zero-mean, complex value Gaussian noise process with variance σ_w^2, and T_s is the sampling interval. The aim of timing synchronization is to estimate the value of $\tilde{d}$. The timing estimation is performed by feeding the received time-domain samples to a sliding window correlator. The output of this correlator is expected to exhibit a peak when the sliding window is perfectly aligned with the received reference (preamble) block. The resulting timing estimate $\tilde{d}$, as given by Morelli *et al*. (2007) is:

$$\tilde{d} = \arg\left(\max_{d} \left\{ \left| M(d) \right| \right\} \right) \tag{4}$$

where, $M(d)$ is the timing metric.

PROPOSED ESTIMATOR

The proposed timing offset estimation method has exploited the basic property of the chirp signal which showed that it peaked in the fractional Fourier domain corresponding to an optimum angle α_{opt}' associated with the FRFT kernel. This particular characteristic of the FRFT on chirp signal has been used for searching the start of training sequence in received signal. Therefore, at the receiver side, the FRFT of auto-correlation of received signal is determined and its peak is observed. Almeida (1994) defines the FRFT of a signal x(t) at an angle α as:

$$\Im[x(t)] = X_\alpha(u) = \int_{-\infty}^{\infty} x(t)\, K_\alpha(t,u)\, dt \tag{5}$$

where, $K_\alpha(t,u)$ is the kernel of FRFT, which is given as:

$$K_\alpha(t,u) = \begin{cases} \sqrt{\dfrac{1-j\cot(\alpha)}{2\pi}}\, e^{(j/2)\left\{(t^2+u^2)\,Cot(\alpha) - 2\,t\,u\,Co\sec(\alpha)\right\}} & ; \quad if\ \alpha \neq n\pi \\ \delta(t-u) & ; \quad if\ \alpha = 2n\pi \\ \delta(t+u) & ; \quad if\ \alpha = (2n+1)\pi \end{cases} \tag{6}$$

In order to make the transform technique compatible with the discrete signal as normally encountered in signal processing applications, the discrete version of the transform technique is needed. Therefore, the sampling type discrete FRFT (DFRFT) algorithm as given by Ozaktas *et al*. (1996) has been considered for the calculation of the FRFT of the signal in the proposed scheme.

The training sequence of proposed method is a chirp signal with chirp rate '2a', duration 'τ' and bandwidth '2a τ', defined as:

$$x(t) = e^{j\,2\,\pi\,(a t^2 + bt)}\ rect\left[t/\tau\right] \tag{7}$$

where, b is the centre frequency of chirp signal. It is known that a finite duration chirp signal can be concentrated maximally in the fractional Fourier domain with an angle $\alpha_{opt,}$ which is determined by the chirp rate '2a'of the signal satisfying the relation as given by (Ozaktas *et al*., 1996; Tao *et al*., 2009):

$$\cot(\alpha_{opt}) = -4\pi a \tag{8}$$

To illustrate this behaviour of chirp signal, a simulation exercise was performed by considering the chirp signal with chirp rate of 16 and unity duration. Thereafter, FRFT of chirp signal at optimum angle $\left(\alpha_{opt} = 3.13165\right)$ and two nearby angles (α = 3.12 and α = 3.15) was determined, as shown in fig.1. It can easily be depicted from the results that at an optimum angle, the FRFT of chirp has a sharp and pronounced peak in comparison to other nearby angles. This conforms to the fact that the FRFT transforms a chirp signal into a delta function at an optimum angle corresponding to the chirp rate associated with the chirp signal.

The correlation theorem for FRFT domain as given by Singh & Saxena (2011) is:

$$r_{xx}(\tau) \;\leftrightarrow\; R_{xx}(u) \tag{9}$$

where, $r_{xx}(\tau)$ is defined as weighted auto-correlation of signal x(t) and $R_{xx}(u)$ is defined as FRFT of $r_{xx}(\tau)$, which are defined as:

$$r_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t+\tau)\; e^{j\,t\,(\tau+t)\,Cot\,(\alpha)}\, dt \tag{10}$$

$$R_{xx}(u) = \sqrt{\frac{2\pi}{1-j\,Cot(\alpha)}}\; e^{\frac{-j}{2}\left\{u^2\,Cot(\alpha)\right\}}\; X_\alpha(-u)\, X_\alpha(u) \tag{11}$$

From (11), it is evident that the FRFT at an angle α of the time-domain weighted auto-correlation ($r_{xx}(\tau)$) is equivalent to the multiplication of signal's FRFT $(X_\alpha(u))$, the mirror image of signal's FRFT$(X_\alpha(-u))$, a chirp function and a scaling factor. The proposed timing estimation method is based on the auto-correlation of received signal. However, the timing metric is generated by using (11). The algorithm to generate timing metric is described in the following paragraph.

A sliding window (N samples) method is used to generate timing metric *M*(*d*). The 'd' is a time index. First, the DFRFT of every N samples of received signal is determined at the optimum angle $\alpha_{opt'}$. Then, this DFRFT is used to calculate the $R_{xx}(u)$ as given in (11). The maximum of $|R_{xx}(u)|$ is used to give the value of timing metric corresponding to index 'd'. This window slides sample by sample as the receiver searches for the training symbol. Therefore, the timing metric of proposed method is defined as:

$$M(d) = \max_{u} \left| \sqrt{\frac{2\pi}{1-j\,Cot(\alpha)}}\; e^{\frac{-j}{2}\left\{u^2\,Cot(\alpha)\right\}} \left(X_{\alpha_{opt},d}(u)\right)^2 \left(X_{\alpha_{opt},d}(-u)\right)^2 \right| \tag{12}$$

where, $X_{\alpha_{opt},d}(u)$ is a N-point DFRFT of *x*(n+d). The maximum of timing metric *M*(*d*) gives the starting point of the training symbol.

The timing metric of the proposed method, under no noise and no channel condition is shown in Fig.2. The 1024 sub-carrier OFDM system with 128 cyclic prefix has been considered for the generation of this timing metric. The correct timing point is indexed as 0 in the figure. The timing metric of algorithms given by Schmidl and Cox (1997), Minn *et al.* (2000) and Park *et al.* (2003) is also shown in Fig.2 for comparison.

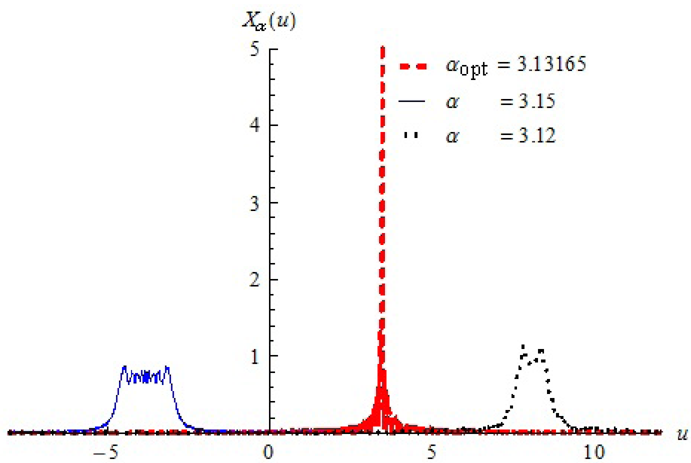

Fig.1: FRFT of a chirp signal (7) for different angle α with a = 8, b=55

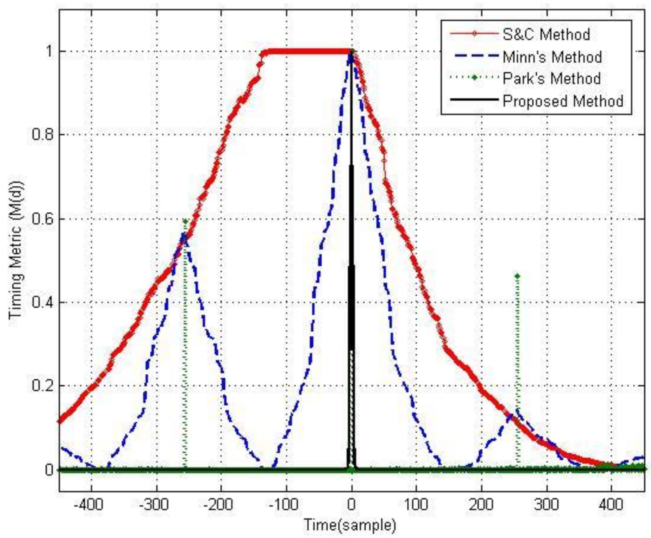

Fig.2: Timing Metric of Different Estimators

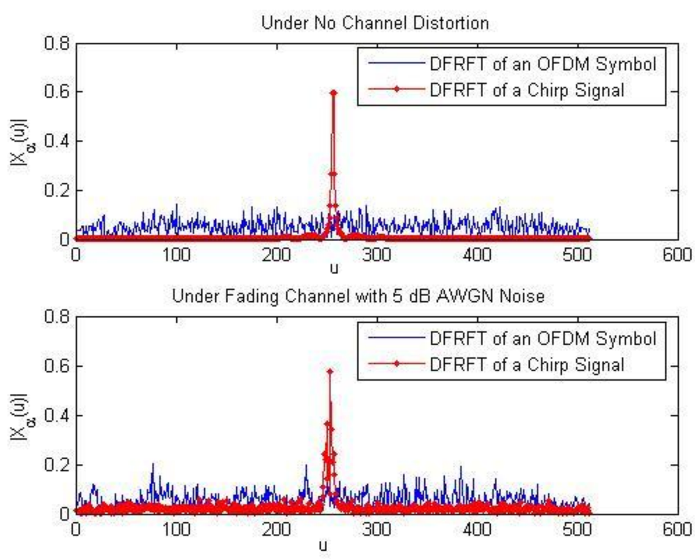

Fig.3: DFRFT of an OFDM and chirp signal

The timing metric of Schmidl and Cox's method has a plateau of length equal to length of cyclic prefix (128 in the present example). This plateau is reduced in Minn's method. The timing metric with Park's method has a sharp peak at correct time but some side peaks are also visible, whereas in the proposed method, the timing metric has a sharp peak with no side peaks and is clearly evident from the plots.

In order to explain the superiority of the proposed timing metric over other included methods in Fig.2, the DFRFT of an OFDM symbol and of a chirp signal under both conditions (no channel distortion and channel distortion with noise) are shown in Fig.3. An OFDM symbol with 512 sub-carriers has been considered for this analysis and a chirp signal defined in (7) is taken with values of ‘a’ = 435 and ‘b’ =76. It is clearly visible from the plots (Fig.3) that the maximum value of DFRFT of chirp signal is much larger than the maximum value of DFRFT of OFDM symbol. This concept is used to search the preamble in received signal.

PERFORMANCE EVALUATION

In this section, the performance of the proposed method is presented and compared with the methods given by Schmidl and Cox (1997), Minn *et al.* (2000), Park *et al.* (2003), Shi and Serpedin (2004) and Awoseyila *et al.* (2008). In this comparison, an OFDM system with 64 sub-carriers and 16 cyclic prefix with QPSK modulation is considered. The HIPERLAN/2 indoor channel model (Channel Models, 1998) is used for simulations. A normalized frequency offset of 0.1 is considered to explore the robustness of proposed method. The mean and mean square error was taken as performance evaluation parameter.

Fig.4 and Fig.5 show the comparison of mean and MSE of timing offset with all four estimators in the ***HIPERLAN/2 indoor channel-A***. It is clearly visible from Fig.4 that the value of mean with proposed method is very low (lies in the range of 2 to 0.1) at all the signal to noise ratios (SNR). The values of MSE of timing offset with proposed method are much better as compared to other estimators, as shown in Fig.5.

Simulation results for advance OFDM system like Wi-Max with 256 sub-carriers are also presented in Fig.6. An ISI channel consisting of L=8 paths with path delays of $m_i = 0,1,\ldots,L-1$ samples and an exponential power delay profile having average power of $e^{-m_i/L}$ has been considered for simulation.

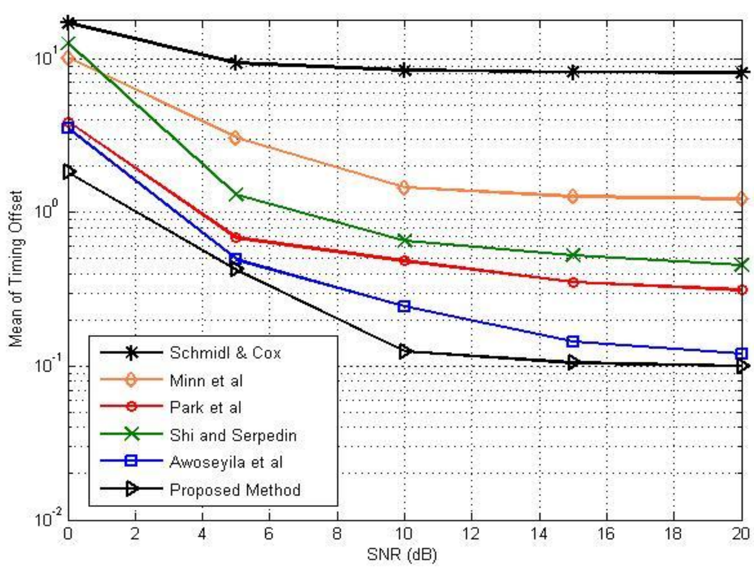

Fig.4: Mean of Timing offset estimation of different methods in HIPERLAN/2 indoor channel-A

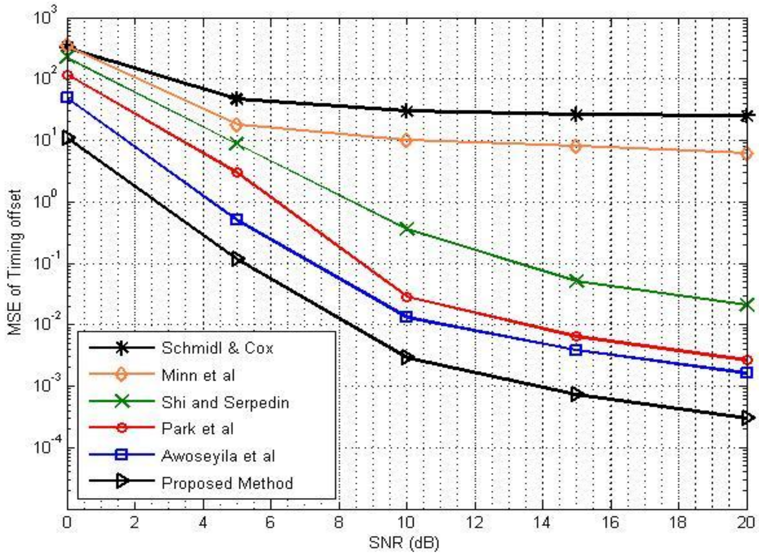

Fig.5: MSE of Timing offset estimation of different methods in HIPERLAN/2 indoor channel-A

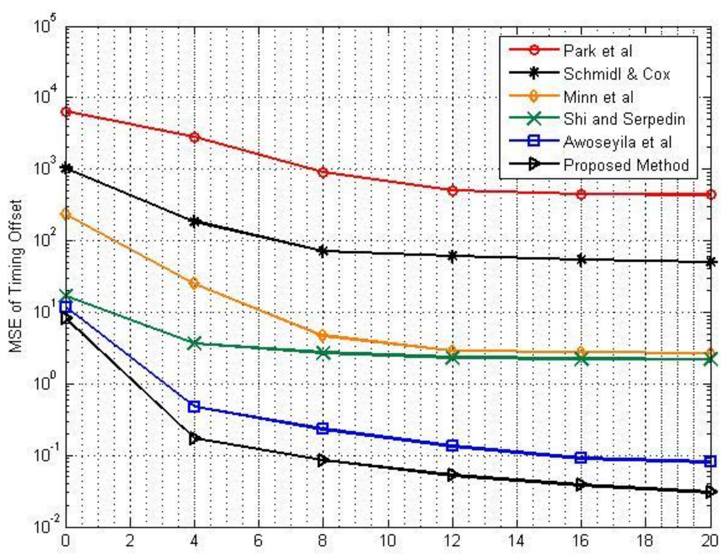

Fig.6: MSE of Timing offset estimation of different methods in Wi-Max system

This improvement in the performance is due to the impulse–shape like timing metric of proposed method in contrast to the timing metric of other methods, as discussed in the previous sections and shown in Fig.2. In addition, as shown in Fig.3, the effect of fading channel and noise is less on chirp signal. The DFRFT of chirp signal gives one sharp peak in fading channel in comparison to OFDM symbol. This concept has been used for generating timing metric. Therefore, the timing offset with proposed timing metric is lesser in both types of channel.

The cost of this improvement is complexity in terms of complex multiplications and additions. The complexity of the proposed method is high in comparison to other methods as shown in Table 1 because other methods generate timing metric by taking auto-correlation in time domain directly whereas the proposed method firstly takes the DFRFT of received signal and then generates timing metric, as given in expression (12).

TABLE 1
Approximate Computational Complexity of Various Estimators

Methods	Complex multiplication	Complex Addition
Schmidl & Cox (S&C,1997)	$N/2$	$N/2-1$
Minn *et al* (2000)	$N/2$	$N/2-1$
Park *et al* (2003)	$N/2+1$	$N/2$
Shi & Serpedin (2004)	$3N/2$	$3N/2-1$
Awoseyila *et al* (2008)	$(7N+8)/4$	$(7N-8)/4$
Proposed	$(N\ \log_2 N)+2N+1$	$N\ \log_2 N$

CONCLUSION

A new method is proposed for timing offset estimation in an OFDM system. The proposed method is based on the localization of chirp signal in the FRFT domain. The suggested algorithm for timing offset estimation in both ***HIPERLAN/2 indoor channel-A*** and in strong fading channel (**Wi-Max** with 256 sub-carriers) provides the lowest MSE when compared to other available methods. However, to obtain such improvements in MSE of timing offset estimation, the computational complexity (in terms of number of complex additions and multiplications) increases. The proposed algorithm of timing offset estimation with better accuracy will certainly improve the performance metric of an OFDM system by way of making the ICI and bit error rate better. This augmentation of performance metric is a better proposition even at the cost of enhanced computational complexity.

REFERENCES

Almeida, L. B. (1994). The fractional Fourier transform and time–frequency representations. *IEEE Trans. Signal Process, 42*(11), 3084–3091.

Awoseyila, A. B., Kasparis, C., & Evans, B.G. (2008). Improved preamble aided timing estimation for OFDM systems. *IEEE Commun. Lett*., *12*(11), 825-827.

Boumard, S., & Mammela, A. (2009). Robust and Accurate Frequency and Timing Synchronization using Chirp Signals. *IEEE Trans. Broadcasting*, *55*(1), 115-123.

Channel Models for HIPERLAN/2 in Different Indoor Scenarios (1998), *ETSI BRAN* 3ERI085B.

Keller, T., & Hanzo, L. (2000). Adaptive multicarrier modulation: A convenient framework for time-frequency processing in wireless communications. *Proc. IEEE*, *88*(5), 611–640.

Minn, H., Zeng, M., & Bhargava, V. K. (2000). On timing offset estimation for OFDM systems. *IEEE Commun. Lett.*, *4*(7), 242–244.

Morelli, M., Kuo, C.-C. J., & Pun, M.-O. (2007). Synchronization Techniques for Orthogonal Frequency Division Multiple Access (OFDMA) A Tutorial Review. *Proc. IEEE, 95*(7), 1394-1427.

Ozaktas, H. M., Arikan, O., Kutay, M.A., & Bozdagi, G. (1996). Digital computation of the fractional Fourier transform. *IEEE Trans. Signal Process, 44*(9), 2141–2150.

Park, B., Cheon, H., Kang, C., & Hong, D. (2003). A Novel Timing Estimation Method for OFDM Systems. *IEEE Commun. Lett., 7*(5), 239-241.

Ren, G., Chang, Y., Zhang, H., & Zhang, H. (2005). Synchronization methods based on a new constant envelope preamble for OFDM systems. *IEEE Trans. Broadcasting*, *51*(1), 139-143.

Schmidl, T. M., & Cox, D. C. (1997). Robust frequency and timing synchronization for OFDM. *IEEE Trans. Commun., 45*(12), 1613–1621.

Sharma, K. K., & Joshi, S. D. (2007). Time delay estimation using fractional Fourier transform. *Signal Processing, 87*(5), 853–865.

Shi, K., & Serpedin, K. (2004). Coarse frame and carrier synchronization of OFDM systems: A new metric and comparison. *IEEE Trans. Wireless Commun., 3*(4), 1271–1284.

Singh, A. K., & Saxena, R. (2011). Correlation theorem for fractional Fourier transform. *International Journal of Signal Processing, Image Processing and Pattern Recognition, 4*(2), 31-39.

Singh, A. K., & Saxena, R. (2011). *On convolution and product theorems for FRFT, Wireless Personal Communications*. DOI 10.1007/s11277-011-0235-5.

Singh, A. K., & Saxena, R. (2011). Recent developments in FRFT, DFRFT with their applications in signal and image processing. *Recent Patents on Engineering*, *5*(2) 113-138.

Sun, H. B., Liu, G. S., Gu, H., & Su, W. M. (2002). Application of the fractional Fourier transform to moving target detection in airborne SAR. *IEEE Trans. on Aerospace and Electronics Systems*, *38*(4), 1416-1424.

Tao, R., Li, X. M., Li, Y., & Wang, Y. (2009). Time-Delay Estimation of Chirp Signals in the Fractional Fourier domain. *IEEE Trans. Signal Process, 57*(7), 2852–2855.